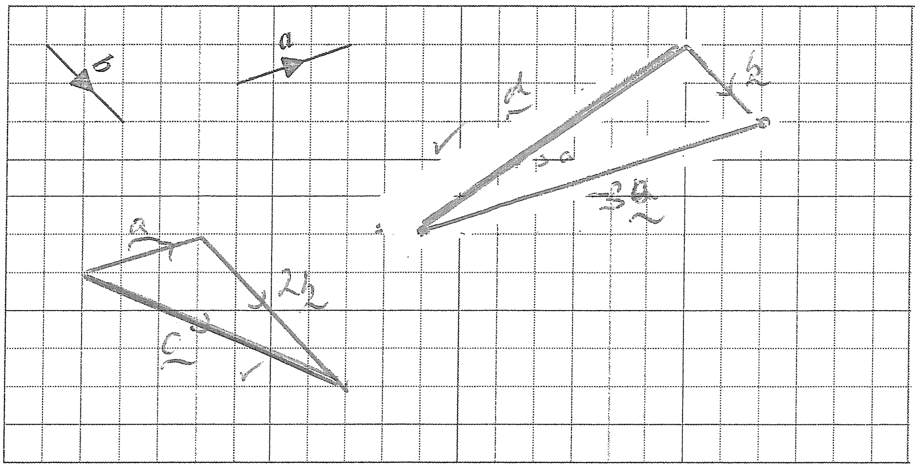
Section One: Calculator-free (52 Marks)

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (7 marks)

(a) Two vectors, **a** and **b** are shown on the grid below.

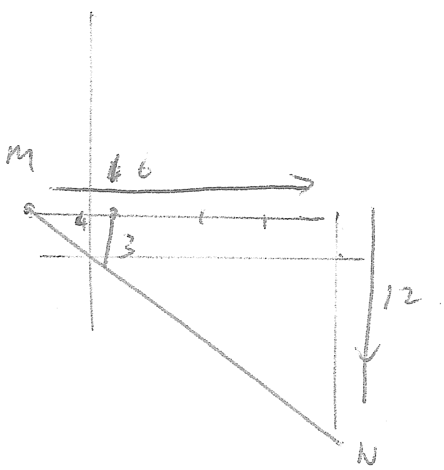


Draw and label the vectors **c** and **d** on the grid, where **c** = **a** + 2**b** and **d** = **b** − 3**a** . (2 marks)

(b) Determine a unit vector perpendicular to the vector $8\mathbf{i} - 6\mathbf{j}$. (2 marks)

$$\frac{6\underset{\sim}{i} + 8\underset{\sim}{j}}{10}$$

(c) The point P divides the line segment from M(−3, 3) to N(13, −9) in the ratio 1:3. Determine the position vector of point P. (3 marks)



$$\overrightarrow{OP} = \frac{3}{4}\langle -3, 3\rangle + \frac{1}{4}\langle 13, -9\rangle$$

$$= \left\langle \frac{-9+13}{4}, \frac{9-9}{4} \right\rangle$$

$$= \langle 1, 0\rangle$$

OR

$\overrightarrow{MN} = \langle 16, -12\rangle$

$\overrightarrow{MP} = \langle 4, -3\rangle$

$\therefore \overrightarrow{OP} = \langle -3, 3\rangle + \langle 4, -3\rangle$

$= \langle 1, 0\rangle$

Question 2 **(6 marks)**

The statement ***'If two rectangles are congruent then they have the same area.'*** is true.

(a) Write the inverse of the statement and explain whether or not the inverse is also true. (2 marks)

If two Rectangles are not Congruent then they do not have same Area

False See Below

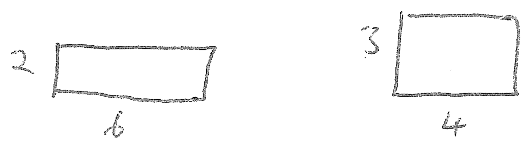
(b) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

Always true.

If two Rectangles do not have same Area Then they are not Congruent.

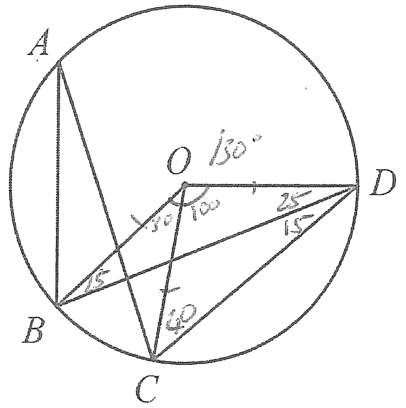
(c) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

If 2 Rectangles have the same Area they are congruent. False



Question 3 **(6 marks)**

(a) In the diagram below, $\angle OBD = 25°$ and $\angle OCD = 40°$.



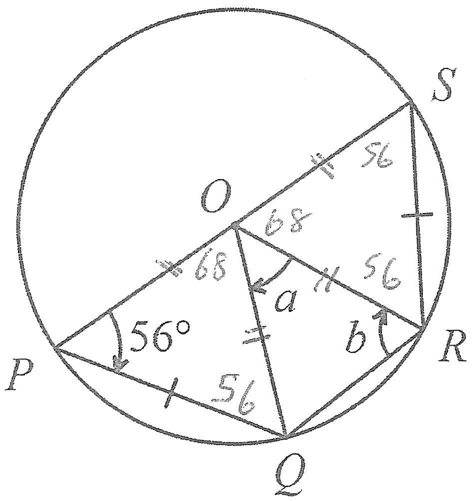
Determine the sizes of

(i) $\angle BDC$. 15° (1 mark)

(ii) $\angle BOC$. 30° (1 mark)

(iii) $\angle CAB$. 15° (1 mark)

(b) Determine, the sizes of the angles marked a and b in the diagram below. Show your reasoning on the diagram. (3 marks)



$a = 180 - 136$

$= \underline{\underline{44}}$

$b = \dfrac{180 - 44}{2}$

$= \underline{68°}$

Question 4 **(8 marks)**

(a) Simplify $\frac{28! \times 7!}{10! \times 26!}$. (2 marks)

$$= \frac{\cancel{28}^{7} \times \cancel{27}^{3} \times \cancel{26!} \times \cancel{7!}}{10 \times \cancel{9} \times \cancel{8}^{2} \times \cancel{7!} \times \cancel{26!}}$$

$$= \frac{21}{20}$$

(b) Prove that ${}^{n}P_{r} = n \times {}^{n-1}P_{r-1}$. (3 marks)

$$RHS = n \times \frac{(n-1)!}{[(n-1)-(r-1)]!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^{n}P_{r}$$

(c) Use the result ${}^{n}P_{r} = n \times {}^{n-1}P_{r-1}$ to determine:

(i) ${}^{9}P_{5}$ given that ${}^{10}P_{6} = 151200$ (1 mark)

$${}^{10}P_{6} = 10 \times {}^{9}P_{5}$$

$$\frac{151200}{10} = {}^{9}P_{5}$$

(ii) ${}^{11}P_{5}$,given that ${}^{9}P_{3} = 504$ (2 marks)

$${}^{11}P_{5} = 11 \times 10 \times {}^{9}P_{3}$$

$$= 5040 \times 11$$

$$= \underline{55440}$$

DO NOT WRITE IN THIS SECTION AS IT WIL BE CUT OFF.

Question 5 **(9 marks)**

The vectors $\mathbf{a}$ and $\mathbf{b}$ are given by $\mathbf{a} = (5,12)$ and $\mathbf{b} = (2,-1)$.

(a) Determine

(i) $\mathbf{a} - 3\mathbf{b}$. (1 mark)

$= \langle 5-6 \;,\; 12+3 \rangle$

$= \langle -1 \;,\; 15 \rangle$

(ii) $|\mathbf{a}| \times |\mathbf{b}|$. (1 mark)

$|a| = \sqrt{169}$ $\quad |b| = \sqrt{5}$

$= 13$

$\therefore \; 13\sqrt{5}$

(iii) the vector projection of $\mathbf{a}$ onto $\mathbf{b}$. (3 marks)

$= \dfrac{a \cdot b}{|b|} \times \dfrac{b}{|b|}$

$= \dfrac{5\times 2 + 12 \times -1}{\sqrt{5}} \cdot \dfrac{\langle 2, -1 \rangle}{\sqrt{5}} = \dfrac{-2}{5}\langle 2, -1 \rangle$

(b) Determine the vectors $\mathbf{c}$ and $\mathbf{d}$ if $2\mathbf{c} - 3\mathbf{d} = \mathbf{a}$ and $(\mathbf{c} - 2\mathbf{d} = 2\mathbf{b}) \times -2$ (4 marks)

$-2c + 4d = -4b$

$d = -4b + a$

$= \langle 8+5 \;,\; 4+12 \rangle$

$= \langle 3 \;,\; 16 \rangle$

$c = 2b + 2d$

$= \langle 4+-6 \;,\; -2+32 \rangle$

$= \langle -2 \;,\; 30 \rangle$

Additional working space.

Question Number: _________

Alternative 5b)

Let $\underset{\sim}{c} = c\underset{\sim}{i} + d\underset{\sim}{j}$ $\quad \underset{\sim}{d} = e\underset{\sim}{i} + f\underset{\sim}{j}$

then $2\underset{\sim}{c} - 3\underset{\sim}{d} = \langle 5, 12 \rangle$ and $\underset{\sim}{c} - 2\underset{\sim}{d} = \langle 4, -2 \rangle$

$\langle 2c-3e, 2d-3f \rangle = \langle 5, 12 \rangle$ and $\langle c-2e, d-2f \rangle = \langle 4, -2 \rangle$

$$\left.\begin{array}{l} 2c-3e = 5 \\ c-2e = 4 \end{array}\right\} \begin{array}{l} ① \\ \times 2 \; ② \end{array} \qquad \left.\begin{array}{l} 2d-3f = 12 \\ d-2f = -2 \end{array}\right\} \begin{array}{l} ① \\ ② \times 2 \end{array}$$

$$\Rightarrow \left.\begin{array}{l} 2c-3e = 5 \\ 2c-4e = 8 \end{array}\right\} \begin{array}{l} ① \\ ②' \end{array} \qquad \Rightarrow \left.\begin{array}{l} 2d-3f = 12 \\ 2d-4f = -4 \end{array}\right\} \begin{array}{l} ① \\ ②' \end{array}$$

$e = -3 \quad$ 1 - 2' $\qquad\qquad f = 16$

$\Rightarrow c = -2$ from ② $\qquad\qquad d = 30$

$\therefore \underset{\sim}{c} = -2\underset{\sim}{i} + 30\underset{\sim}{j} \qquad \underset{\sim}{d} = -3\underset{\sim}{i} + 16\underset{\sim}{j}$

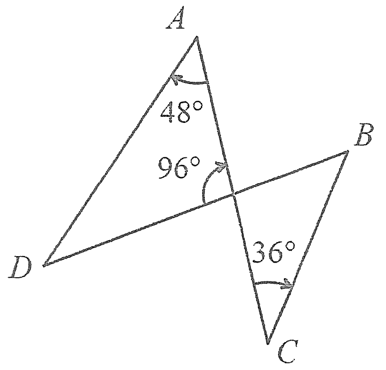
DO NOT WRITE IN THIS SECTION AS IT WIL BE CUT OFF.

Additional working space.

Question Number: ________

Question 6 **(7 marks)**

(a) Prove that it is possible to draw a circle through the points A, B, C and D shown below. (Assume A and B are on a circle and show that C and D must also be on the same circle.) (3 marks)



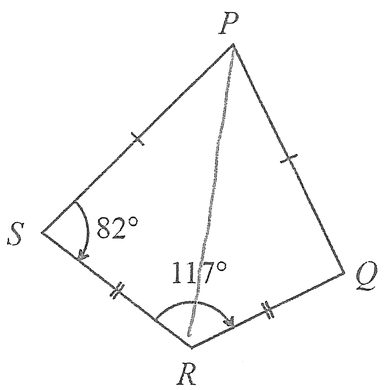
$s\angle ADB = 180 - 48 - 96$ ($180°$ in $\triangle$)
$= 36°$

$\therefore s\angle ADB = s\angle ACD$

$\therefore$ CD are on the Same Arc

So ABCD are on a circle.

(b) Prove by contradiction that it is impossible to draw a circle through the vertices of the quadrilateral shown below. (4 marks)



Assume PSQR is Cyclic.

$\Rightarrow s\angle S + s\angle Q = 180°$

But $\triangle PSR \cong \triangle PQR$ (SSS)

($PS = PQ$ given
$SR = RQ$ given
$PR = RP$ Same)

$\therefore s\angle S = s\angle Q = 82°$

But $82° + 82° \neq 180$

$\therefore$ Not on Same Circle.

Accepted PSRQ is a Kite if given Statements to justify it

DO NOT WRITE IN THIS SECTION AS IT WILL BE CUT OFF.

Question 7 **(9 marks)**

(a) A bag contains 17 identical cubes except for their 3 different colours. There are four coloured orange, six coloured blue and seven coloured white.

(i) How many different arrangements of coloured cubes are possible when three cubes are drawn from the bag and placed in a line? (1 mark)

3 colours $\therefore$ $3 \times 3 \times 3 = \underline{\underline{27}}$

(ii) How many different combinations of coloured cubes are possible when three cubes are drawn from the bag? (3 marks)

O O O   O O B/W   O B W   Reasoning needed
B B B +   B B O/W +
W W W   W W O/B

$3 + 6 + 1 = 10$

(iii) Determine the least number of cubes that should be removed from the bag to ensure that the resulting selection contains at least three cubes of one colour. Justify your answer. (2 marks)

By Pigeon Hole need 2 in each colour +1

$= \underline{7}$

(b) Determine how many integers are needed {1, 2, 3, ..., ~~98~~ 100}, to ensure we have a pair that sum to 99 (3 marks)

1, 98
2, 97
⋮
49, 50

There are 49 Possible pairs
By Pigeon Hole if we have 49 integers from those pairs then we need 3 more (99, 100 + a match) to ensure a pair.

$\therefore \underline{52}$

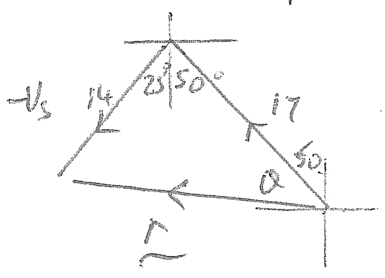
END OF QUESTIONS

DO NOT WRITE IN THIS SECTION AS IT WIL BE CUT OFF.

(c) A small ship is travelling with a constant speed of 14 knots on a bearing of 025° and another, larger ship is travelling with a constant speed of 17 knots on a bearing of 310°.

Determine the velocity of the large ship relative to the small ship. (5 marks)

$_LV_S = V_L - V_S$ $\Longrightarrow$



$|_LV_S| = \sqrt{14^2+17^2-2(14)(17)\cos 75°}$

$= \sqrt{361.8}$

$= 19.02$ knots (Accept from g c)

$\sin\theta = \dfrac{14\sin 75}{19.02}$

$= 45.3°$ (Accept from g c)

Bearing $= 310 - 45°$

$= 265°$ T

Alternative: $V_L = \langle 17\cos 140, 17\sin 140\rangle$

$V_S = \langle 14\cos 65, 14\sin 65\rangle$

$V_L - V_S = \langle -18.94, -1.76\rangle$ $\theta = 270 - \tan^{-1}\left(\frac{-1.76}{-18.94}\right)$

$= 265°$

$|_LV_S| = 19.02$ kn.

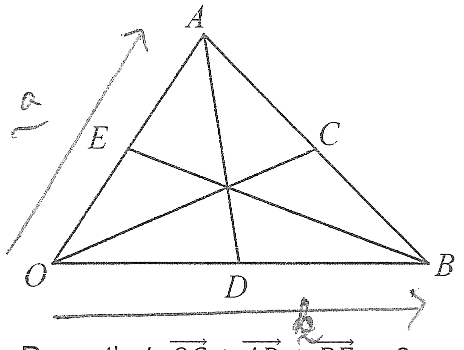
Must give in form it started

Question 16 **(9 marks)**

The medians of triangle OAB are OC, AD and BE, as shown below.

(A median joins a vertex to the midpoint of the opposite side of the triangle).

Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$



(a) Prove that $\overrightarrow{OC} + \overrightarrow{AD} + \overrightarrow{BE} = \mathbf{0}$. (4 marks)

$\overrightarrow{OC} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$

$\overrightarrow{AD} = -\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$

$\overrightarrow{BE} = -\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$

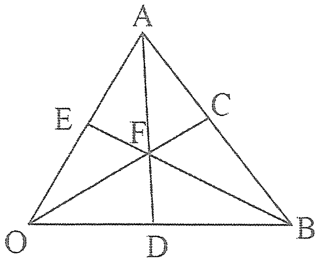
$\therefore \overrightarrow{OC} + \overrightarrow{AD} + \overrightarrow{BE}$

$= \frac{1}{2}\underset{\sim}{a} - \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{b} - \underset{\sim}{b}$

$= \underset{\sim}{0}$.

DO NOT WRITE IN THIS SECTION AS IT WILL BE CUT OFF.

(b) The centroid, F, is the point of intersection of the medians.



Determine $\overrightarrow{AF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ ($\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$). (5 marks)

(*Hint: Let, $\overrightarrow{EF} = h\overrightarrow{EB}$, $\overrightarrow{OF} = k\overrightarrow{OC}$ and first solve for h and k*)

$\overrightarrow{EF} = h\,\overrightarrow{EB}$

$= h\left[-\frac{1}{2}\underset{\sim}{a} + \underset{\sim}{b}\right]$

$\overrightarrow{OF} = k\,\overrightarrow{OC}$

$= k\left[\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}\right]$

$\therefore \overrightarrow{AF} = \overrightarrow{AE} + \overrightarrow{EF}$ or $\overrightarrow{AO} + \overrightarrow{OF}$

$\Rightarrow \frac{-a}{2} + \frac{-ha}{2} + h\underset{\sim}{b} = -\underset{\sim}{a} + \frac{k}{2}\underset{\sim}{a} + \frac{k}{2}\underset{\sim}{b}$

$\Rightarrow \frac{-(h+1)}{2} = \frac{k-2}{2}$ (A) and $h = \frac{k}{2}$ (B)

Equate '$\underset{\sim}{a}$' Equate '$\underset{\sim}{b}$'

Ok from Calculator

So $\frac{-\left(\frac{k}{2}+1\right)}{\cancel{2}} = \frac{k-2}{\cancel{2}}$ Sub (B) into (A)

$1 = \frac{3k}{2}$

$\frac{2}{3} = k$ so $h = \frac{1}{3}$

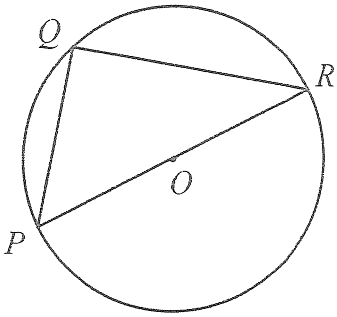
$\therefore AF = -\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$

$= \frac{1}{3}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a}$

Question 17 **(8 marks)**

(a) The diagram shows a triangle with vertices P, Q and R that lie on a circle with centre O. Chord PR passes through O. Prove, by contradiction, that angle QPR is acute.

(4 marks)



assume s$\angle QPR$ is ≥ 90

then as s$PQR = 90$ (Angle in Semicircle)

and s$\angle PQR$ + s$\angle RPQ$ + s$\angle QPR$

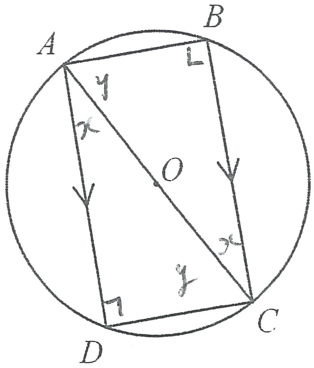
$= 90 + > 0 + \geq 90$

> 180!! Impossible.

$\therefore \angle QPR$ is < 90

DO NOT WRITE IN THIS SECTION AS IT WIL BE CUT OFF

(b) In the diagram below, O is the centre of the circle on which points A, B, C and D lie. Chord AC passes through O and BC is parallel to AD
Prove that the quadrilateral $ABCD$ is a rectangle. (4 marks)



$s\angle ABC = s\angle ADC = 90°$ (Angles in Semi Circle)

$s\angle BCA = s\angle DAC = x$ (Alternate ∠'s)

$\therefore s\angle ACD = s\angle BAC = 180 - 90 - x = y$

as 180° in each $\triangle$

ie $x + y = 90°$.

$\therefore s\angle DAB = s BCD = 90° = x + y$

$\therefore$ 4 Right Angles a Rectangle

OR

$s\angle ABC = s\angle ADC = 90°$

$s\angle BCA = s\angle DAC = x$ (Alternate Angles in ∥ lines)

$AC = CA$ Same Side.

$\therefore \triangle ABC \cong \triangle CDA$ (AA Corr Side).

$\therefore AB = CD$ and $DC = DA$

So $ABCD$ is Parallelogram with 90°

$\therefore$ a Rectangle

Question 18 **(8 marks)**

(a) A small coach has 24 seats, arranged in six rows of four seats each, with two seats in each row on either side of the central aisle. A group of passengers consisting of ten males and nine females board the bus.

(i) Determine how many combinations of empty seats are possible once everyone has sat down. (1 mark)

$^{24}C_{19} = {}^{24}C_5 = \underline{42504}$

(ii) How many fewer combinations are there if the females all sit on one side of the aisle and the males all sit on the other side? (3 marks)

$\overset{M}{^{12}C_{10}} \cdot \overset{F}{^{12}C_9} * 2$ (Left or Right) $= 29\ 040$

$\therefore$ $\underline{13464 \text{ Less}}$

(b) Determine the number of possible four letter permutations of the letters of the word RELOAD. (1 mark)

$^6P_4 = 360$

(c) Determine the number of **different** 8 letter words possible from the letters of the word RELOADED. (2 marks)

$\frac{8!}{2!\,2!} = 10080$

(d) How many of the words in part (d) have the word "deed" within them? (1 mark)
Eg. "RODEEDAL"

$5! = 120$

DO NOT WRITE IN THIS SECTION AS IT WIL BE CUT OFF.

Question 19 **(7 marks)**

A small boat has to travel across a river from A to B, where $OA = 60\mathbf{i} + 35\mathbf{j}$ metres and $OB = 356\mathbf{i} - 125\mathbf{j}$ metres. A uniform current of $-1.5\mathbf{i} + 2.5\mathbf{j}$ m/s is flowing in the river and the boat can maintain a steady speed of 4 m/s.

(a) Determine, in the form $a\mathbf{i} + b\mathbf{j}$, the velocity vector the small boat should set to travel directly from A to B. *(Warning: Your calculator may take quite a while to solve the necessary equations, don't panic!)* (5 marks)

$\overrightarrow{AB} = 296\underset{\sim}{i} - 160\underset{\sim}{j}$

$\therefore \langle 296, -160 \rangle = t\langle a, b \rangle + t\langle -1.5, 2.5 \rangle$

$\therefore 296 = t(a - 1.5)$ EQN 1 $\quad -160 = t(b + 2.5)$ EQN 2

and $a^2 + b^2 = 4^2$ EQN 3

From Calc, very slowly ~~a = −3.9736, b = 0.4587~~

$a = 2.56 \quad b = -3.073$

OR $t = \dfrac{296}{a-1.5} = \dfrac{-160}{b+2.5}$ and $a^2+b^2=16$.

then Sub for a or b

note cannot have $a = -3.9736$ as this makes time negative!!

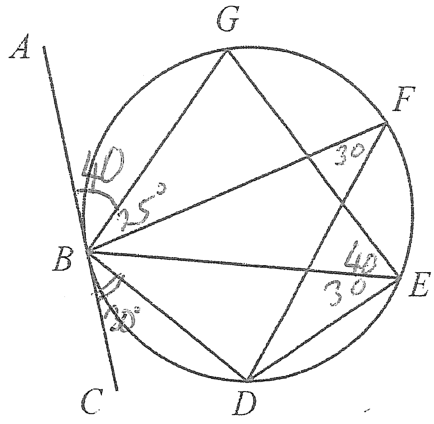
(b) Calculate, to the nearest minute and second, how long the journey will take. (2 marks)

$\dfrac{-160}{b+2.5} = t = \dfrac{296}{a-1.5} = \underline{4 \text{ min } 39 \text{ s}}$

$= \underline{279\text{s}}$

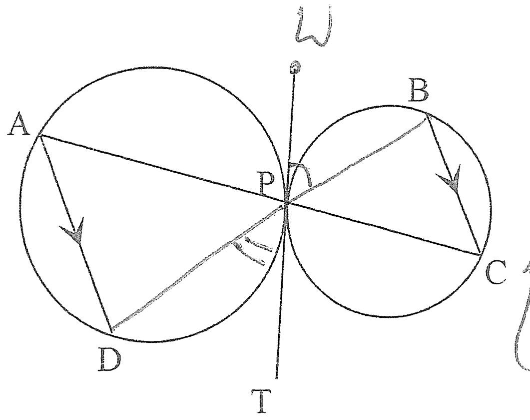
Question 20 **(7 marks)**

(a) In the diagram below, AC is a tangent to the circle at B. If $\angle ABG = 40°$, $\angle GBF = 25°$ and $\angle BFD = 30°$, determine the size of angle DBF.
Clearly show your reasoning by filling in values on the diagram. (3 marks)



$DBF = 180 - 40 - 25 - 30$
$= \underline{85°}$

(b) In the diagram below, the line AC passes through the point P, where both circles touch each other. The line AD is parallel to line BC. TP is tangential to both circles at P.
Prove that the points B, P and D are collinear. (4 marks)



$s\angle BCP = s\angle DAP$ (Alternate)

$s\angle WPB = s\ PCB$
$s\angle TPD = s\angle DAP$
Angle in Alternate Segment = Angle to the tangent

$\therefore s\angle WPB = s\angle TPD$

So DPB is a straight line as Vertically Opp $\angle$'s are congruent and TPW is straight.

Note Many Variations are possible

END OF QUESTIONS

Section Two: Calculator-assumed (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 (5 marks)

Three vectors are given by $\mathbf{a} = 7\mathbf{i}$, $\mathbf{b} = 6\mathbf{i} + 9\mathbf{j}$ and $\mathbf{c} = x\mathbf{i} - 5\mathbf{j}$.

(a) Use your calculator to determine the angle between $\mathbf{a}$ and $\mathbf{b}$, to the nearest degree. (2 marks)

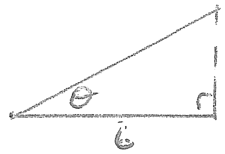
$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$= \frac{42}{7\sqrt{36+81}}$$

$$\underline{\theta = 56^\circ}$$

No penalty not nearest degree.

OR



as $\underset{\sim}{a} = 7\underset{\sim}{i}$

$\tan^{-1}(1.5) = 56^\circ$

(b) Determine all possible values of x if $\mathbf{a} + \mathbf{c}$ and $\mathbf{b} + \mathbf{c}$ are perpendicular. (3 marks)

$\underset{\sim}{a} + \underset{\sim}{c} = \langle 7+x, -5 \rangle$

$\underset{\sim}{b} + \underset{\sim}{c} = \langle 6+x, 4 \rangle$

$\therefore \langle \underset{\sim}{a} + \underset{\sim}{c} \rangle \cdot \langle \underset{\sim}{b} + \underset{\sim}{c} \rangle = 0$

$\Rightarrow (7+x)(6+x) - 20 = 0$

From Calc.

$\underline{x = -11 \text{ or } -2}$

OR $(\underset{\sim}{a} + \underset{\sim}{c})(\underset{\sim}{b} + \underset{\sim}{c}) = 0$

$\Rightarrow \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c} + \underset{\sim}{c} \cdot \underset{\sim}{b} + c^2 = 0$

$42 + 7x + 6x - 45 + x^2 + 25 = 0$

$x^2 + 13x + 22 = 0$

$\underline{x = -2 \text{ or } -11}$

Question 9 **(8 marks)**

(a) A multiple choice test has twelve questions and each question has three possible choices. If all questions are attempted, in how many ways can the test be answered? (2 marks)

$3^{12} = 531441$

(b) A set S contains all the integers between 3 and 102 inclusive. Determine

(i) how many numbers in set S are multiples of 7. (1 mark)

$102 \div 7 = 14.57 \Rightarrow \underline{14}$

(ii) how many numbers in set S are multiples of 3 or 7. (2 marks)

$102 \div 3 = 34$

$102 \div 21 = 4.8 \therefore$ 4 Repeats.

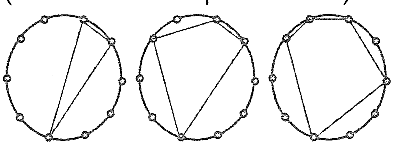
$3 \text{ or } 7 = 34 + 14 - 4$

$= \underline{44}$

(iii) how many numbers in set S are multiples of either 3 or 7 but not both. (1 mark)

$44 - 4 = 40$

(c) Ten points are equally spaced around the circumference of a circle.
Determine the number of simple (non-self-intersecting) convex polygons that can be formed by joining either three, four or five of these points with straight line segments (as in the examples below). (2 marks)



3 points 4 points 5 points

$\binom{10}{3} + \binom{10}{4} + \binom{10}{5}$

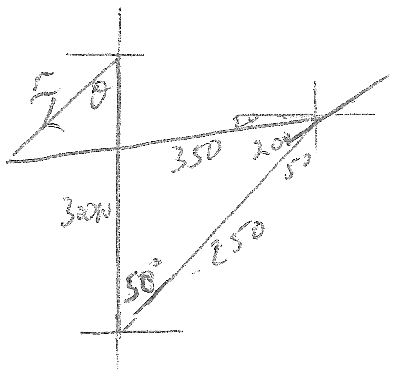
$= 582$

Question 10 **(7 marks)**

Three forces are applied to a body. One has magnitude 300 N and acts due south. Another has magnitude 250 N and acts on a bearing of 050°.

(a) If the third force has a magnitude of 350 N and acts on a bearing of 250°, determine the magnitude and direction of the resultant force. (6 marks)

Using CAS Calculator geom.



$|r| = 293N$

$\theta = 27.94^\circ$

$\therefore$ 293 N on 208°.

OR $F_1 = 0\underset{\sim}{i} - 300\underset{\sim}{j}$

$F_2 = 250\cos 40\underset{\sim}{i} + 250\sin 40\underset{\sim}{j}$

$\left(= 250\sin 50\underset{\sim}{i} + 250\cos 50\underset{\sim}{j}\right)$

$F_3 = 350\cos 200\underset{\sim}{i} + 350\sin 200\underset{\sim}{j}$

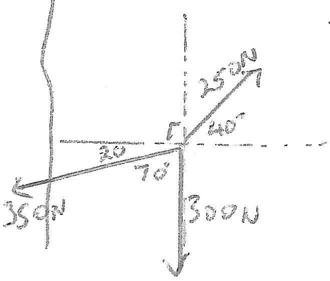
$= -350\cos 20\underset{\sim}{i} - 350\sin 20\underset{\sim}{j}$

$\underset{\sim}{r} = -137.38\underset{\sim}{i} - 259.01\underset{\sim}{j}$

$|\underset{\sim}{r}| = 293.2N$

$\text{Direction} = 270 - \tan^{-1}\left(\frac{259}{137.38}\right)$

$= \underline{208^\circ}$



(b) A 4th force is applied so that the body is in equilibrium. Use your answer from part a) to write down the magnitude and direction of the 4th force. (1 mark)

293.2 N on 28°T (Follow through)

Question 11 **(6 marks)**

(a) A triangle PQR has vertices $P(1, 1)$, $Q(5, 3)$ and $R(3, 7)$. Determine the vector $\overrightarrow{QM}$, where M is the midpoint of side PR. (3 marks)

$\overrightarrow{QM} = \frac{1}{2}\left[\overrightarrow{QP} + \overrightarrow{QR}\right]$

$= \frac{1}{2}\left[(-4,-2) + (-2,4)\right]$

$= \langle -3, 1 \rangle$

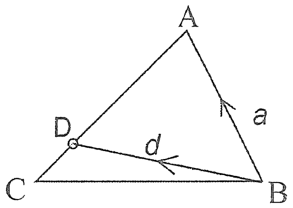
(using Q as Origin)

OR $\overrightarrow{QP} + \frac{1}{2}\overrightarrow{PR}$

$= \langle -4,-2 \rangle + \langle 1, 3 \rangle$

$= \langle -3, 1 \rangle$

(b) ABC is a triangle with point D on side AC such that $AD = \frac{3}{4}AC$. If $\overrightarrow{BA} = \boldsymbol{a}$ and $\overrightarrow{BD} = \boldsymbol{d}$, show that $\overrightarrow{BC} = \frac{1}{3}(4\boldsymbol{d} - \boldsymbol{a})$. (3 marks)



OR $\overrightarrow{BC} = \overrightarrow{BD} + \frac{1}{3}\overrightarrow{AD}$

$= \underset{\sim}{d} + \frac{1}{3}(\underset{\sim}{d} - \underset{\sim}{a})$

$= \frac{4}{3}\underset{\sim}{d} - \frac{1}{3}\underset{\sim}{a}$

$= \frac{1}{3}(4\underset{\sim}{d} - \underset{\sim}{a})$

$\overrightarrow{AC} = \frac{4}{3}(\underset{\sim}{d} - \underset{\sim}{a})$

$\overrightarrow{BC} = \underbrace{\underset{\sim}{a}}_{\overrightarrow{BA}} + \underbrace{\frac{4}{3}(\underset{\sim}{d} - \underset{\sim}{a})}_{\overrightarrow{BC}}$

$= \frac{4}{3}\underset{\sim}{d} + \frac{3}{3}\underset{\sim}{a} - \frac{4}{3}\underset{\sim}{a}$

$= \frac{1}{3}(4\underset{\sim}{d} - \underset{\sim}{a})$

Question 12 **(7 marks)**

(a) Vectors $\mathbf{a}$ and $\mathbf{b}$ have the same magnitude and vectors $\mathbf{a}$ and $\mathbf{c}$ are perpendicular, where $\mathbf{a} = \begin{bmatrix} m \\ n \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Determine the values of m and n. (3 marks)

$|a| = |b|$

$\Rightarrow m^2 + n^2 = (-4)^2 + 6^2$

$\underline{m^2 + n^2 = 52}$

$a \cdot c = 0$

$\Rightarrow 2m + 3n = 0$

From CAS $m = -6, n = 4$ OR $m = 6, n = -4$

(b) Determine the scalar projection of a velocity of 12 m/s on a bearing of 65° onto a velocity of 20 m/s on a bearing of 280°, giving your answer to three significant figures. (2 marks)

$\text{Proj } a/b = |a| \cos\theta$

$= 12 \cos(215° \text{ or } 145°)$

$= \underline{-9.83 \text{ m/s}}$

(c) The work done, in joules, by a force of $\mathbf{F}$ Newtons in changing the displacement of an object by $\mathbf{s}$ metres is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$.

A force acting on a bearing of 160° does work of 1 200 joules. If the object moved a distance of 350 cm on a bearing of 135°, determine the magnitude of the force. (2 marks)

$\underline{3.5\text{m}}$

$F \cdot S = |F||s| \cos 25$

$\dfrac{1200}{3.5 \cos 25°} = |F|$

$|F| = 378\text{N}$

(3.78N 1 mark only)

Question 13 **(8 marks)**

(a) A triangle has vertices at $A(-3,\ 1)$, $B(-1,\ 4)$ and $C(5,\ 0)$.

(i) Determine the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{BC}$. (2 marks)

$\overrightarrow{AB} = \langle 2, 3 \rangle \quad \overrightarrow{AC} = \langle 8, -1 \rangle \quad \overrightarrow{BC} = \langle 6, -4 \rangle$

-1 / Error.

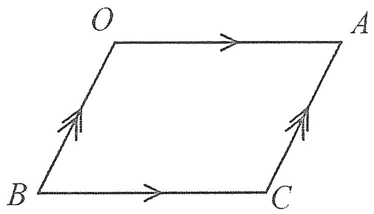
(ii) Use a vector method to prove that triangle ABC is right-angled. (2 marks)

$\overrightarrow{AB} \cdot \overrightarrow{BC}$

$= \langle 2 \times 6 + 3 \times -4 \rangle$

$= \underline{0}$ $\quad \therefore \perp$ at B

(b) Use a vector method to prove that if the diagonals of a parallelogram are perpendicular to each other, then the parallelogram is a rhombus. (4 marks)



Let $\overrightarrow{OA} = \underset{\sim}{a} = \overrightarrow{BC}$ as //ogram.

$\overrightarrow{OB} = \underset{\sim}{b} = \overrightarrow{AC}$ as parallelogram.

Most left out, didn't penalise but should do so!

Then $\overrightarrow{OC} = \underset{\sim}{a} + \underset{\sim}{b}$

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$ as $\perp$ Diagonals.

$\Rightarrow \quad (\underset{\sim}{a} + \underset{\sim}{b})(\underset{\sim}{b} - \underset{\sim}{a}) = 0$

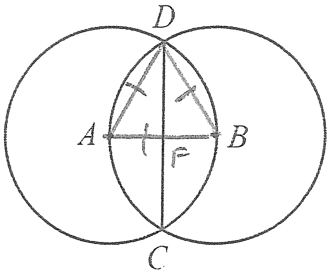
$\underset{\sim}{a} \cdot \underset{\sim}{b} - a^2 + b^2 - \underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$b^2 - a = 0 \quad \therefore |a| = |b|$

It is a Rhombus

Question 14 **(9 marks)**

(a) Two circles of radius 12.6 cm, with centres A and B as shown below, have a common chord CD. Determine, with justification, the length CD. (3 marks)



Either use $\triangle ABC$ is Equilateral

$\therefore \frac{DF}{DB} = \frac{\sqrt{3}}{2}$

$DF = 12.6 \times \frac{\sqrt{3}}{2}$

$DC = 2DF = \underline{21.82}$

or Cosine Rule

$DC^2 = DB^2 + BC^2 - 2DB \cdot BC \cos 120°$

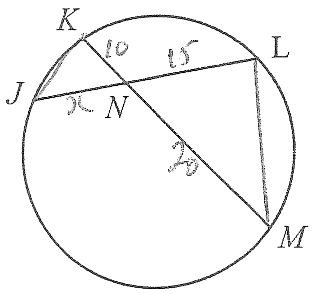
$= 12.6^2 + 12.6^2 - 2(12.6)(12.6)\frac{-1}{2}$

$= 3(12.6)^2$

$DC = \sqrt{3} \times 12.6$

$= \underline{21.82}$.

(b) In the diagram below, $KN = 10$ cm, $LN = 15$ cm and $MN = 20$ cm. Determine, with justification, the exact length of JN. (3 marks)



Could use Similar Δ's to get

$\frac{10}{15} = \frac{x}{20}$

$x = 13\frac{1}{3}$

OR Chord Theorem

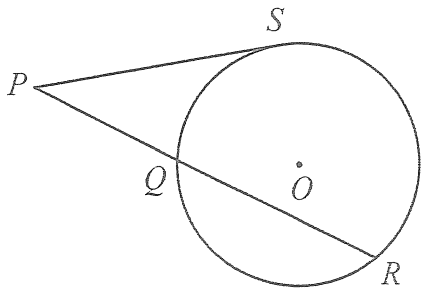
$KN \times MN = JN \times LN$

$10 \times 20 = 15x$

$\frac{200}{15} = x$

$\underline{13\frac{1}{3} = x}$

(c) Determine the length PQ, if that the length of chord QR is 10.5 cm and the length of the tangent PS is 9.5 cm. (3 marks)



$PS^2 = PQ \times PR$

$9.5^2 = PQ(PQ + QR)$

$9.5^2 = x(x + 10.5)$

$x = \cancel{-16.10}$ or $\underline{5.604}$

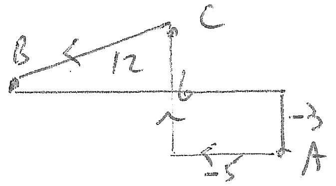
-1 If don't Exclude -16.1

DO NOT WRITE IN THIS SECTION AS IT WIL BE CUT OFF.

Question 15 **(9 marks)**

(a) A small body A has position (12, -3) m relative to another small body B. If a third small body C has position (-5, 6) relative to A, determine the position of B relative to C.

(2 marks)



$_B r_C = \langle -7, -3 \rangle$

OR $_A r_B = \langle 12, -3 \rangle$

$\Rightarrow {_B r_A} = \langle -12, 3 \rangle$ $(\overrightarrow{AB})$

$_C r_A = \langle -5, 6 \rangle$ $(\overrightarrow{AC})$

$\therefore \overrightarrow{CB} = \langle -12, 3 \rangle - \langle -5, 6 \rangle$

$_B r_C = \underline{\langle -7, -3 \rangle}$

(b) To a cyclist moving with velocity (21, −5) km/h the wind appears to have velocity (−9, 3) km/h. Determine the true speed of the wind. (2 marks)

$_W V_C = V_W - V_C$

$V_W = {_W V_C} + V_C$

$= \langle 12, -2 \rangle$

$|V_W| = \sqrt{148}$

$= 12.165$ km/h